Computational Analysis of A Non-Newtonian Fluid Past Obstacles of Altered Shapes

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Abstract

The computational study of non-Newtonian fluid such as paint past a solid hurdle in channel flow is substantial to many manufacturing problems related with industries. Numerous techniques such as analytical and numerical simulation can be used to study such fluid flow. In the current study shear thinning viscoelastic fluid is taken into consideration. 2-D study is under investigation in the current study using Oldroyld-B model.

In the contemporary study the core focus is emphasized on flow of Non-Newtonian fluid past different shapes of solid obstacles, such as triangle and trapezium. Structured meshes are used to investigate the accuracy, stability and consistency of the numerical scheme for both types of obstacles. The parabolic behaviour of the flow verifies stability and consistency of the scheme. Finite element Semi implicit time-stepping Taylor/ Galerkin Pressure Correction Scheme is employed for the simulation. The main focus of the study is to analyse the flow pattern of the fluids while crossing solid hurdles because of its importance in mixing and beading process of materials.

Keywords: Solid hurdles, Finite Element, Simulation, Oldroyld B model

INTRODUCTION

The applications of CFD plays vital role in industrial problems, especially the problems related with mixing of fluids and beading of materials. From the literature it has been analysed that mixing of various fluids have direct effect on the quality of production. The flow behaviour of Oldroyld B fluid has been investigated by [1]. For Industrial problems, flow past a solid is significant apart from type of fluid. In the applications various fluids are found like viscous, viscoelastic etc, containing flows in the region of solid obstacles such as triangle or trapezium. Shape of product may comprise solid obstacles to form a finished product; nevertheless moulding improves design and functionality of the product. From the study of Chan [2], it is critically observed that very negligible difference occurs in terms of velocity, pressure and wall shear stresses for flow of fluid in channel. It is very complex to simulate viscoelastic fluids for the asymmetrical solid surfaces because of sudden change of the geometry with singularities may be the main reason of very soaring levels of stresses, and in turn improve numerical instabilities for a viscoelastic fluid and lead to divergence at very immense Deborah numbers [1]. The mirror method has been proposed for numerical results for the flow past obstacle by [3]. Cruz *et al*. [4], described analytical solutions for fully developed flow. Flow of fluid through composite obstacles (stenosis) is highlighted by Ellahi [5]. The effects of obstacles in channel (artery) have been investigated by [6] using power law and further shear stress effects are also investigated in their study.

PROBLEM DESCRIPTION

Two solid hurdles with different ratios on the inner surface of straight channel are considered for the study. Meshes with different elements are designed to analyse the results because in the application the mesh generation plays vital role for the stability and consistency of results (Figure 1 & 2).

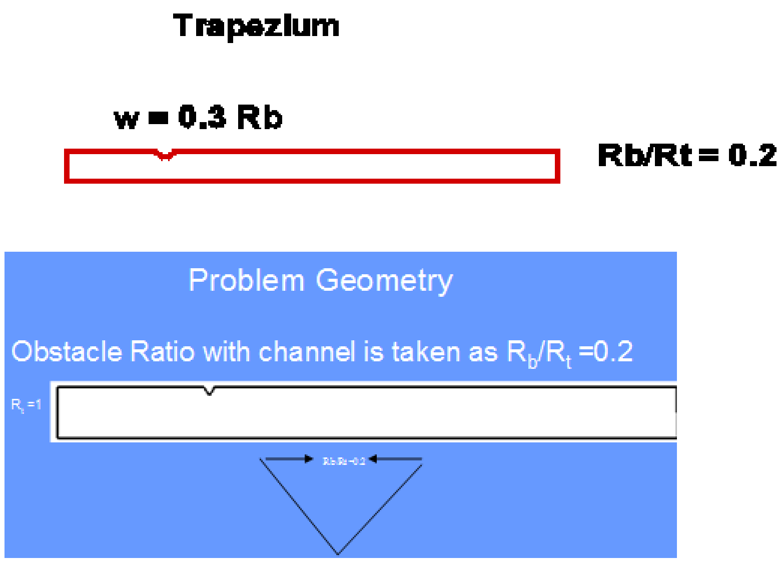

Figure 1: Problem Geometry (Triangular)

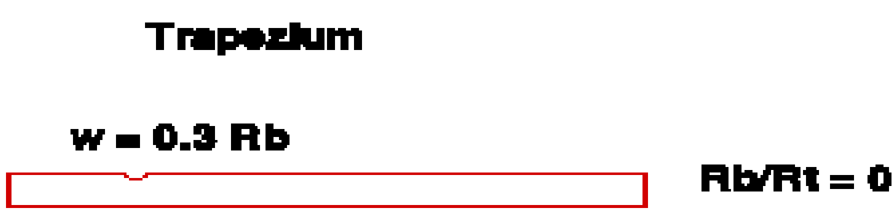

Figure 2: Problem Geometry (Trapezium)

Inlet Boundary Conditions

Inlet conditions for the flow of fluid in channel are expressed as

X=X max [1-(y/Y)^2]

Here y is distance of point along Y-axis and Y is radius and equal to 1.

Therefore $X_{max} = (1 - y^2)$

Outlet Boundary Conditions

Boundary conditions at the outlet can be expressed as $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0$ 0, and also No-slip boundary conditions are imposed on the wall [3].

Governing Equations

Momentum, continuity and stress constitutive equations are used to obtain system of linear equations for the simulation.

$\rho \frac{\partial u}{\partial t} = \nabla \sigma - \rho u.\nabla u,$ where σ is cauchy stress tensor and is defined as,

$\nabla.u = 0$

$\sigma = -pI + T$, T is total stress tensor, given by

$T = -\tau + 2\mu_2 D,$ μ_2 is a solvent viscosity and τ is the polynmeric extra stress contribution and D is a deformation tensor, given by

$D = \left(L + L^t\right)/2\,, L = \nabla u$

Oldroyld-B model is considered for the computation of extra stress

$\tau + \lambda_1 \tau^{\nabla} = 2\mu\left(D + \lambda_2 D^{\nabla}\right),$ *where upper convected is defined as*

$\tau^{\nabla} = \partial\tau / \partial t + u.\nabla\tau - \nabla u.\tau - \tau.\nabla u^T$

u is velocity vectors and τ are the shear stresses

Numerical Algorithm

$$A_{ij} = \int_{\Omega} \phi_i \phi_j d\Omega \text{ , } B(U)_{ij} = \int_{\Omega} \phi_i \phi_j U 1 \nabla \phi_j d\Omega, \ (Ck)_{ij} = \int_{\Omega} \psi_i \frac{\nabla \phi_j}{\nabla x_k} \ d\Omega,$$

$$D_{ij} = \int_{\Omega} \nabla \psi_i \nabla \psi_j \ d\Omega, \text{ and } F_{ij} = \int_{\Omega} \nabla \phi_i \nabla \phi_j \ d\Omega$$

Where, consistent mass matrix, diffusive matrix ,convection matrix ,pressure-gradient matrix, pressure stiffness matrix denoted by A,B,C(U),D, and F, respectively.

To find an approximate solution, a semi-implicit Taylor-Galerkin/Pressure-Correction method is used through Fortran Code. System of linear equations is solved by using iterative Jacobi's method and for the pressure terms Choleski's method is preferred as from the literature.

RESULTS AND DISCUSSION

In the present study, solid hurdles of triangular and trapezium during the flow of fluids are considered for numerical investigation. Flow of fluid past an asymmetrical triangle and trapezium hurdles are considered for the study. For the obstacle of triangular asymmetrical type 1500 elements and 3175 nodes are used whereas for trapezium hurdle with 1516 elements and 3209 nodes (Figure 6) are considered for the investigation. Blockage ratio of 1/5 is taken as proposed by [3].Velocity and pressure profiles (Figure 4, 7) for both hurdles are taken using Numerical Schemes. It has been observed that Velocity profile on inlet and outlet of the channel is parabolic (Figure 3) which guarantees the accuracy, consistency and stability of the Numerical scheme. For both meshes maximum pressure is seen at symmetry line, maximum pressure is also observed on the hurdle, however very slight increase in pressure is noticed for the hurdle of trapezium shape (Figure 7).

Computational results are generated using triangular and trapezium shape of obstacles using mesh size of 10x75 .

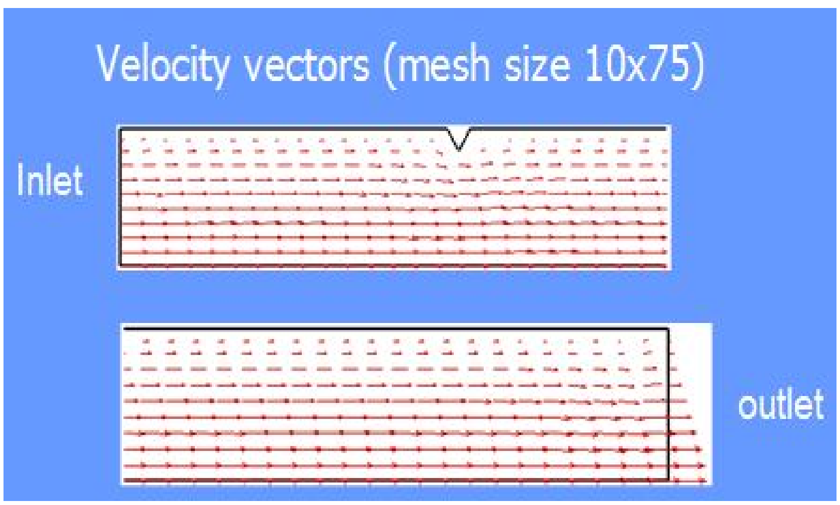

Figure 3: Velocity Profile

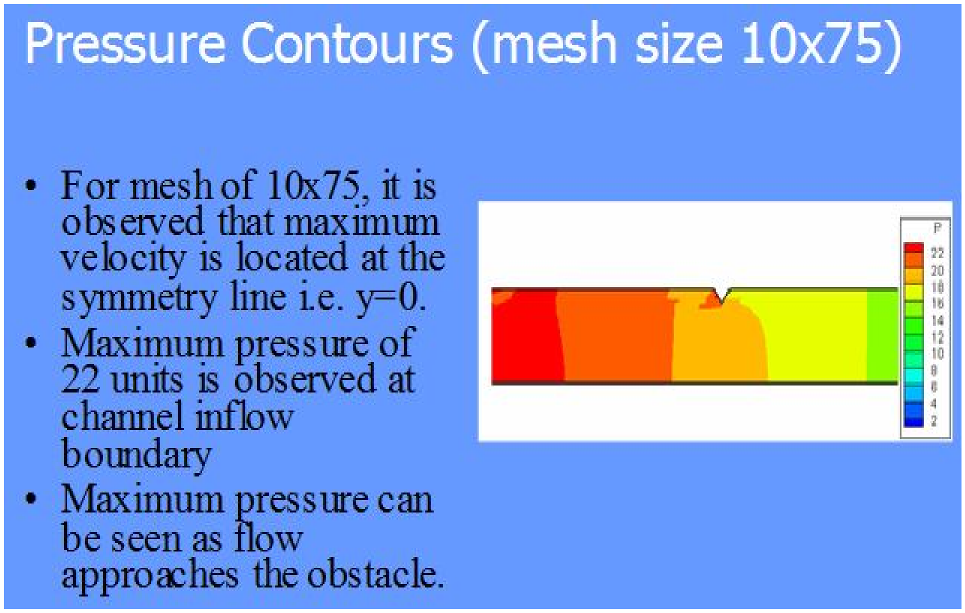

Figure 4: Pressure Contours

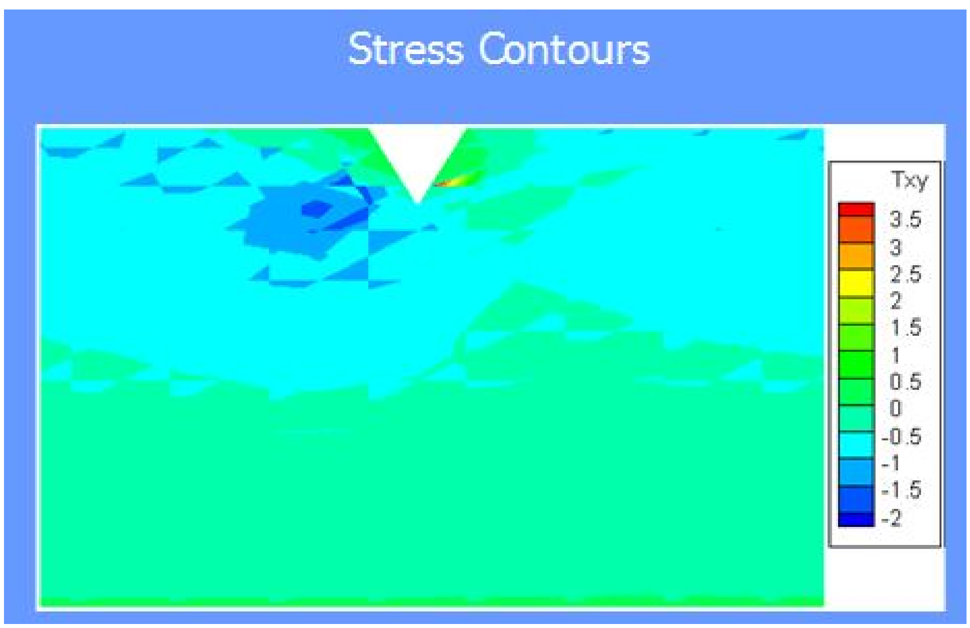

Figure 5: Stress Contour

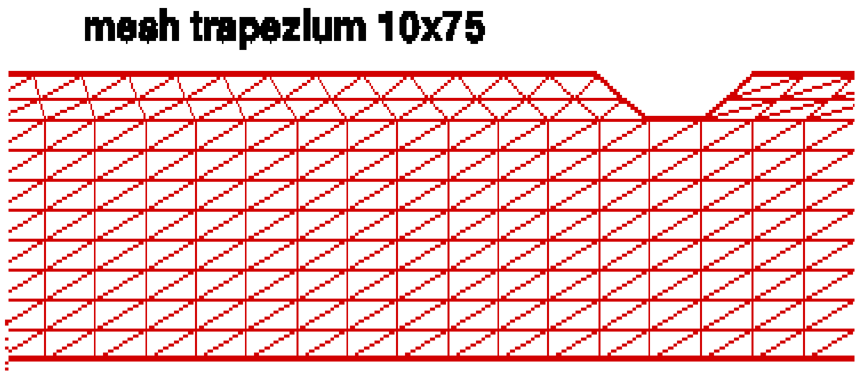

Figure 6: Mesh trapezium

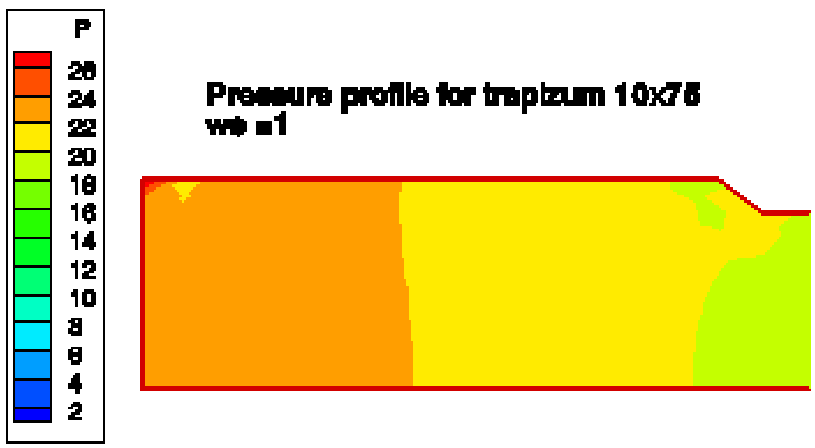

Figure 7: Pressure profile

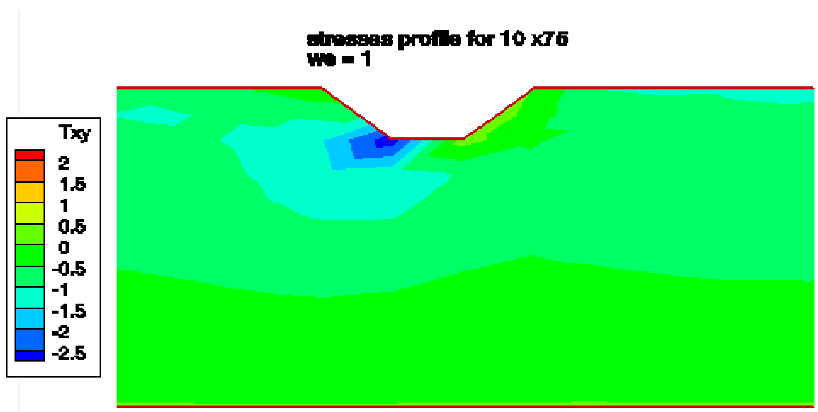

Figure 8: Stresses Profile

CONCLUSION

From the numerical results, it has_been observed that the maximum shear stresses occurs across the obstacle for both meshes with different elements. However, the magnitude of shear stress computed using Triangle mesh is almost double (Figure 5) of the Trapezium mesh (Figure 8) because of narrow space in triangular mesh, since the accurate measurement of shear stress is of importance in mixing flows problems. Hence, it has been concluded from the present study that triangular obstacle capture the solution with good accuracy.

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